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An Assessment of the Appropriate Size Measure for Probability Proportional to Size Path Sampling of Apple Trees

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An Assessment of the Appropriate Size Measure for Probability Proportional to Size Path Sampling of Apple Trees, by Darla DeJong, Nationwide Insurance Company, Columbus, Ohio¹. Mike Fleming, William C. Iwig, and Charles R. Perry Survey Research Branch, Research Division, National Agricultural Statistics Service, United States Department of Agriculture, Washington, DC 20250-2000. March 1995. Research Report No. SRB-95-01.

ABSTRACT

This study evaluates the optimal branch size measure for use in probability proportional to size (PPS) sampling of apple tree branches to estimate the number of apples on a tree. The analysis is based on data collected in the 1991 Apple Objective Yield Pilot Study conducted in Washington. Current NASS Objective Yield procedures for sampling orange, tart cherry, and other fruit trees select branches for fruit counts using probability proportional to size procedures where the size variable is the cross sectional area (CSA) of the branches. Analysis indicates that the CSA is in fact the appropriate size measure to use in PPS sampling of an apple tree, applied in a multiple stage random path approach, regardless of the tree's variety, rootstock, age, or geographic region. PPS sampling of branches based on CSA also provides smaller sampling variances than simple random sampling (SRS) of branches. A cost benefit analysis needs to be conducted to provide final recommendations on an optimal within tree sampling approach for apples.

KEY WORDS

Apple Objective Yield; PPS Sampling; Fruit Count Estimation; Random Path Sampling.

This paper was prepared for limited distribution to the research community outside the U. S. Department of Agriculture. The views expressed herein are not necessarily those of NASS or USDA.

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¹Darla DeJong was on the staff of the Ohio Research Unit at the time of the study and preliminary analysis.

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SUMMARY

This study evaluates branch size measures of the form $x_i = r^{\gamma}$ where r =radius of branch i and $\gamma \geq 0$ for use in probability proportional to size (PPS) sampling of apple tree branches to estimate the number of apples on a tree. Analysis is based on data collected in the 1991 Apple Objective Yield Pilot Study conducted in Washington. A total of 51 trees were used in this pilot study and they were completely mapped and enumerated, which entailed the following steps: the cross sectional area (CSA) of each branch including the stem of the tree was measured; each branch was labeled with an identification number; a schematic sketch was made of the branching structure of each tree for later use as a check on the accuracy of the data. Current NASS objective yield within tree sampling procedures for orange, tart cherry, and other fruit trees select branches for fruit counts using probability proportional to size procedures where the size variable is the cross sectional area of the branches. The PPS sampling procedures are applied in a multiple stage random path approach, where a new branch is selected at each successive fork as long as two or more branches at a fork are larger than an established cutoff CSA. In order to see whether the PPS sampling method is significantly affected by variety, rootstock, age, or region, the trees were grouped accordingly and the CV's were calculated. Analysis indicates that the CSA is in fact the appropriate size measure to use in PPS sampling of an apple tree regardless of variety, rootstock, age, or geographic region. PPS sampling of branches based on CSA provided smaller sampling variances than simple random sampling (SRS) of branches regardless of how the trees were grouped. However, PPS sampling requires additional time for measuring and selecting branches at each stage. A cost benefit analysis needs to be conducted to determine the optimal within tree sampling procedure. The cost analysis would provide recommendations regarding:

- 1) PPS sampling vs. simple random sampling
- 2) the optimal cutoff CSA
- 3) the number of branches to sample at each stage.

INTRODUCTION

Presently, apple production forecasts are made by the Washington Agricultural Statistics Service by means of non-probability surveys and administrative data. As a result, the July forecast has ranged from as high as 22% above to as low as 32% below the final estimate. National Agricultural Statistics Service (NASS) State Statistical Offices (SSO) in Florida, California, Oregon, and Michigan currently conduct fruit and nut tree objective yield surveys based on probability survey methods.

To determine optimal survey methods for apples, a pilot study was conducted during September and October 1991. The Washington State Statistical Office selected 51 trees in the Yakima, Wenatchee, and Columbia Basin regions. The sample consisted of many varieties, rootstocks, and ages of apple trees. Each tree was completely mapped and enumerated. The objective of the pilot study was to evaluate within tree sampling procedures relative to the expected coefficient of variation (CV) and determine whether certain characteristics such as rootstock, variety, age, or geographic region significantly affect the efficiency of the sampling process. Specifically, this initial study evaluates the optimal branch size measure to use in probability proportional to size sampling.

TERMINOLOGY

Probability proportional to size (PPS) sampling has been used by NASS in several forms to estimate the number of fruit on orange, tart cherry, hazelnut, and other fruit trees. Typically, the size variable that has

been used is the cross sectional area (CSA) of the branch, which is proportional to the radius squared ($CSA = \pi r^2$). Other size measures could also be used. Branches are selected proportional to their size beginning at the stem of the tree until the enumerator reaches the outermost branches or until the CSA is less than a pre-described cutoff.

At each stage of PPS path sampling, one branch is selected at random according to its relative size to the other branches at that fork to which it is attached. This selection process, which begins at the stem and proceeds along a path to the terminal branch of a tree, consists of *multiple stages* where each stage corresponds to a fork in the path. With each successive stage, the CSA's become smaller. The process of selecting branches ends when an enumerator reaches that outer limb whose CSA still exceeds a certain prescribed *cutoff* CSA but where every succeeding limb has a CSA less than the cutoff CSA. Thus, the number of stages in a multiple stage process is dependent on the cutoff CSA: the larger the cutoff CSA, the fewer the number of stages. That last limb in the selection process which meets this cutoff criterion is called a *terminal* branch. A *primary* branch is a limb which exceeds the cutoff CSA, is attached to the stem of the tree, and supports at least two terminal branches. Those *intermediate* branches which connect a primary branch with a terminal branch may or may not have fruit. If there is fruit on an intermediate branch, it is called *path fruit*.

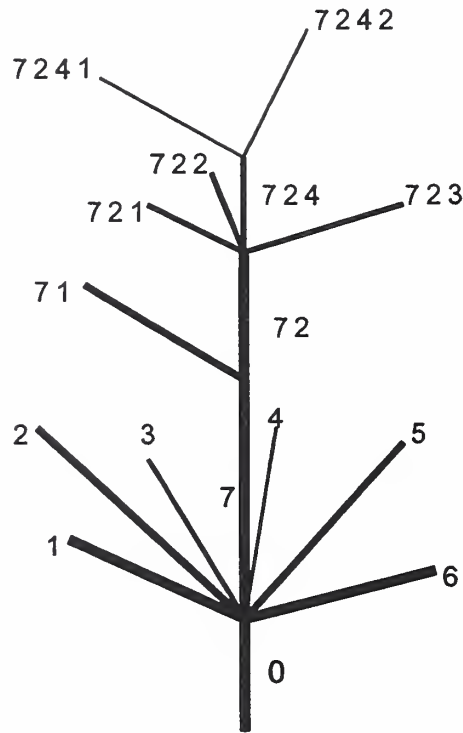
The method of accommodating the path fruit in the estimation process will be discussed later. However, in simplest terms, if the intermediate branches have no fruit, then the

estimate of the quantity of fruit on a tree

(tree load) is $\frac{y_t}{\pi_t}$, where y_t is the number

of fruit on the terminal branch and all others that it supports and π_t is the probability of

selected at the second fork would be counted regardless of the size of the succeeding branches. This approach was evaluated as an option that would simplify data collection, since it involves less within tree sampling. For any particular tree, it is the same as multiple stage sampling when the



Schematic Drawing of the Redchief Enumerated in Table 1.

selecting that terminal branch. For each branch selection scheme, there is a different π_t .

We will look at the probability proportional to size and simple random sampling branch selection schemes. *Two stage* sampling is the same as the multiple stage sampling except that the process does not go beyond the second fork along the path. Consequently, all apples on the branch

cutoff CSA is large enough that there is no more sampling of branches after the second fork.

Simple random sampling (SRS) is a method of sampling where the probability of selecting a branch at a fork is uniformly distributed. SRS can be viewed as PPS sampling where the size measure is the radius to the zero power. Then the size measure is the same for all branches and the

selection reverts to random sampling. It has the attractive feature that it is simpler to use and to teach than the PPS method.

Jessen [2] developed the multiple stage method of PPS sampling based on CSA in the mid-1950's for use in the Orange Objective Yield Survey. It is the same method that is still being used by NASS. A comparison of various sampling plans for estimating the tree load of apple trees is discussed in Houseman [1]. Of the eleven plans that he considered, the PPS method of the kind developed by Jessen and evaluated in this pilot study was the one that Houseman deemed best because it had a relatively small coefficient of variation (CV) and required less labor than the next best alternative (which involved identifying all terminal branches on each tree). However, Houseman did not specifically evaluate alternative size measures or evaluate the approach over a wide range of conditions. This study evaluates those issues and also evaluates their application in two stage sampling.

DATA

The trees which were used in this pilot study were completely mapped and enumerated (see descriptive information in Table A1 of the Appendix). That entailed the following: the CSA of each branch including the stem of the tree was measured; each branch was labeled with an identification number; a schematic sketch was made of the branching structure of each tree for later use as a check on the accuracy of the data.

However, before the analysis began, two problems with the data had to be resolved.

1. A smaller CSA was sometimes reported for a parent branch than a subsequent branch. It is reasonable to assume that the branches become smaller the further one goes down a path though the opposite might occur if an irregularity in the shape of a branches affects an enumerator's measurement. To make the size of the branches decrease along a path, a correction was made to the inconsistent branch to make it arbitrarily 0.1 in² smaller than its parent. There were 9 out of 2,094 branches that had to be corrected this way.

2. Missing data occurred when some terminal branches were beyond the reach of an enumerator. In this case, the average CSA between the parent and the cutoff CSA of 0.5 in² was used as the imputed value.

A cutoff CSA of 0.5 in² was used in designating terminal branches. A schematic diagram of a seven year old Redchief apple tree was shown previously. It illustrates the naming convention that was used to identify the branches. The concept of *depth* facilitated matters when writing the computer programs; it is defined as the number of digits in the identification number with the proviso that the depth of the stem is zero. It can be viewed as a measure of how far removed a branch is from the stem.

To illustrate what the raw data looked like, Table 1 contains the data for the Redchief apple tree whose schematic drawing is shown in Figure 1.

Since the cutoff CSA determines the smallest branch which an enumerator may select, a small cutoff CSA would mean that a large number of branches will be sampled but with fewer apples per terminal branch,

Table 1
Raw Data for a Mapped Tree

Redchief Age: 7 Years Total Fruit: 203					
Branch Identification Number	Depth	Number of Apples on Branch	Number of Apples Supported by Branch	Cross Sectional Area	Type of Branch *
0	0	2	203	7.6 in ²	S
1	1	19	19	1.5	T
2	1	20	20	1.7	T
3	1	24	24	2.4	T
4	1	28	28	2.2	T
5	1	9	9	1.0	T
6	1	5	5	1.7	T
7	1	9	96	4.2	P
71	2	11	11	1.3	T
72	2	2	76	3.8	I
721	3	10	10	0.8	T
722	3	10	10	0.8	T
723	3	0	0	1.0	T
724	3	6	54	2.3	I
7241	4	2	2	1.2	T
7242	4	46	46	1.9	T

* S = Stem P = Primary I = Intermediate T = Terminal

whereas a large cutoff CSA would mean fewer branches sampled but more apples per terminal branch. The number of apples that an enumerator can be expected to count depends on the cutoff CSA and therefore is related to within sampling cost.

To evaluate the effect of the cutoff CSA on the estimate, all mappings of the 51 trees were modified and re-mapped as if a different cutoff CSA had been used in the field.

The original cutoff CSA that was employed for the pilot study was 0.5 in². The procedure for modifying the mapping of a tree involved a computer program which sorted the branches by CSA in ascending order and then kept all branches which had a CSA greater than or equal to a new but larger cutoff CSA. For each new cutoff CSA, the branching structure of the tree's crown had to be reconfigured, in order to make it conform with how the tree would

have been mapped with the new cutoff CSA. With a larger cutoff CSA, there are fewer limbs that could be branches, so that the resulting reconfigured schematic of the crown would appear as if it had collapsed to something smaller. With each successive collapse of the tree by the computer corresponding to a larger cutoff CSA, the system of branching became simpler until at the end of the process the map of the fully collapsed tree consisted of only the stem and two branches. Special attention focused on the process of reconfiguring the map to ensure that after a collapse the new crown consisted of intermediate branches which split at least twice and that the assignment of apples in the new branching structure was correct.

ANALYSIS METHODS

The general form of the multiple stage estimator of the tree load can be written as:

$$\hat{y} = \sum_i^t \left(\frac{y_{k_i}}{\pi_{k_i}} \right) \quad (1)$$

where π_{k_i} is the probability of selecting branch i along the k^{th} path by using either PPS or SRS methods and y_{k_i} is the number of apples on the same branch and along the same path to terminal branch t . Each path ends at a unique terminal branch; therefore, for each tree there is a unique one-to-one correspondence between the inventory of terminal branches and the inventory of paths. If there is no path fruit, then (1)

reduces to the same $\frac{y_t}{\pi_t}$ as referred to in the terminology section.

For the Redchief data shown in Table 1, the path, k , consisting of the branches, i : $\{0, 7, 72, 724, 7242\}$ would have the corresponding apple count, $y_{k_i} : \{2, 9, 2, 6, 46\}$.

The basis of the PPS method is selecting one branch at a fork according to its relative size with respect to the other branches at that fork. Size variables that historically have been used for within tree sampling are all functions of the limb radii at the branch points. For example, PPS sampling using limb cross sectional area as the size variable is equivalent to PPS sampling using limb radius squared as the size variable. Simple random sampling is equivalent to PPS sampling using limb radius to the zero power. For this study, we considered size variables of the form:

$$x_i = r^\gamma \quad (2)$$

where r is the radius of branch i and $\gamma \geq 0$.

The probability of selecting branch i at a fork is:

$$p_i = \frac{x_i}{\sum_{\substack{\text{branches } m \\ \text{at fork}}} x_m} \quad (3)$$

Next the probability of selecting some branch along a path is the product of the p_j 's at each fork along the way from the stem of the tree to the branch of concern, that is, for branch i along the k^{th} path:

$$\pi_{k_i} = \prod_{j=0}^i p_j \quad (4)$$

When $j = 0$, the path section referred to is the part of the tree between the ground and below the first stage branches. When $j=1$,

Table 2
Probabilities of Selection Along a Path for the Redchief

Branch i	PPS to CSA		SRS	
	P_j	π_{k_i}	P_j	π_{k_i}
0	1	1	1	1
7	$\frac{4.2}{1.5+1.7+2.4+2.2+1.0+1.7+4.2} = .2857$.2857	$\frac{1}{7}$	$\frac{1}{7}$
72	$\frac{3.8}{1.3+3.8} = .7451$.2129	$\frac{1}{2}$	$\frac{1}{14}$
724	$\frac{2.3}{.8+.8+1.0+2.3} = .4694$.0999	$\frac{1}{4}$	$\frac{1}{56}$
7242	$\frac{1.9}{1.2+1.9} = .6129$.0612	$\frac{1}{2}$	$\frac{1}{112}$

the path section referred to is the part of the tree on path k above the first stage branch and below the second stage branches and so on.

Let π_{k_i} denote the probability of selecting the terminal branch on path k, then the expected value of the multiple stage estimator, \hat{y} , is:

$$E[\hat{y}] = \sum_{\text{paths } k} \pi_{k_i} \hat{y}_{(k)} \quad (5)$$

where $\hat{y}_{(k)}$ denotes the realization of \hat{y} when path k is selected. By using induction to evaluate the summation, the expected value of $\hat{y}_{(k)}$ is exactly equal to the number of apples on the tree. The two stage estimator is just a special case of the multiple stage one. Therefore, the PPS and SRS multiple stage or two stage estimators

are unbiased.

Using the Redchief tree as in the previous example, Table 2 shows the P_j and π_{k_i} along path k for PPS based on CSA and SRS. Substituting the values of π_{k_i} from Table 2 for PPS sampling into (1), we get an estimate for the number of apples on the Redchief in our example:

$$\hat{y}_{PPS} = \frac{2}{1} + \frac{9}{.2857} + \frac{2}{.2129} + \frac{6}{.0999} + \frac{46}{.0612} = 854$$

The actual tree load is 203, so by using our chosen path the estimate exceeds the actual value by 651 apples. Similarly an estimate of the tree load via SRS is:

$$\hat{y}_{SRS} = \frac{2}{1} + \frac{9}{\frac{1}{7}} + \frac{2}{\frac{1}{14}} + \frac{6}{\frac{1}{56}} + \frac{46}{\frac{1}{112}} = 5,581$$

Again the estimate exceeds the actual number of apples. However, the estimates from paths leading to other terminal branches would be low. As mentioned previously, overall the estimator is unbiased.

The size variable of the form r^γ that provides the optimal PPS multiple stage estimator is obtained by minimizing the variance in (6) over the general class of size variables given in (2).

$$var(\hat{y}) = \sum_{paths\ k} \pi_{k_i} \{\hat{y}_k - Y\}^2 \quad (6)$$

The variance of \hat{y} is a function of γ defined in (2) and the cutoff CSA. Since every tree had been completely enumerated, the variance can be computed for any given γ and cutoff CSA.

The analysis for this pilot study consisted of grouping trees according to types of rootstock, variety, geographic region, and age and then comparing the CV's of the estimates for PPS and SRS methods for both multiple stage and two stage approaches. In order to find a size variable that could be used over all 51 trees or over subgroups of them, a general CV was computed for each group, cutoff CSA, and sampling procedure.

$$CV = \frac{\sqrt{\sum_{trees} sse_i}}{total\ apples\ in\ group} \quad (7)$$

The sum of squared errors (SSE) of the estimated apple load of a tree were summed at regularly spaced cutoff CSA's over groupings of trees to get the CV at a particular γ and cutoff for that grouping. The sse for a particular tree is defined as:

$$sse_i = \sum_k (\hat{y}_{i(k)} - Y_i)^2$$

where $\hat{y}_{i(k)}$ = estimated number of apples on tree i from path k and Y_i = observed number of apples on tree i. Three dimensional graphs of this general CV fitted against the radius power (γ) and the cutoff CSA over all 51 trees were reviewed to determine the optimal γ for multiple and two stage approaches. In addition, the trees were grouped according to variety, rootstock, age, and region. The three dimensional graphs were reviewed to evaluate the effect of the different factors.

Groupings of trees were as follows:

1. Varieties.
 - a. Group 1 consisted of all varieties from the Red Delicious strain.
 - b. Group 2 consisted of all the other varieties.
2. Rootstock.
 - a. Standard.
 - b. Semi-dwarf.
3. Age.
 - a. 1-10 years.
 - b. 11-20 years.
 - c. Over 20 years.
4. Region.
 - a. Yakima.
 - b. Columbia Basin
 - c. Wenatchee

Figure 1: Multiple Stage for All Trees

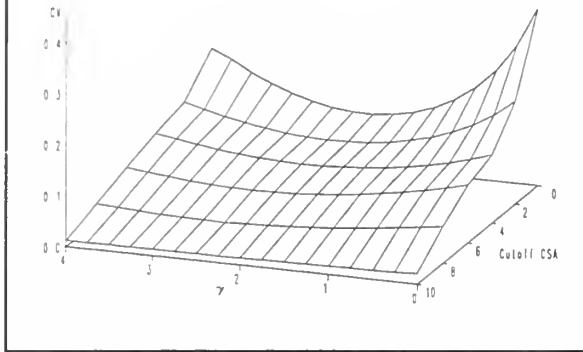
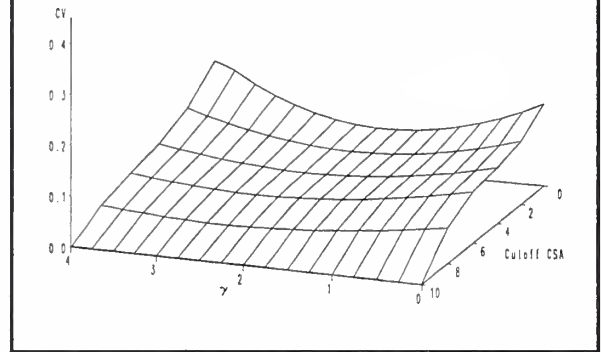


Figure 2: Two Stage for All Trees



RESULTS

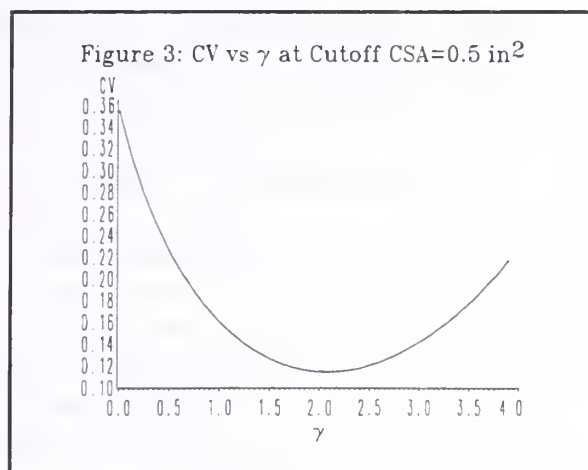
Figures 1 and 2 are three dimensional graphs showing the general CV (Equation 7) as a function of the cutoff CSA and the radius power (γ), based on data from all 51 trees in the study. The figures indicate that for both the multiple stage and two stage approaches, a value for γ of 2 corresponds to a minimum in the CV, regardless of the cutoff CSA. Figure 3 is a cross section of the surface shown in Figure 1 along the smallest cutoff CSA, 0.5 in² and parallel to the γ axis. It clearly shows that using a γ near 2 corresponds to the minimum CV. An optimal γ of 2 implies that the CSA ($=\pi r^2$) is the optimal size variable for probability proportional to size sampling of apple trees. This finding is also consistent for all of the groupings of trees evaluated in the study, as indicated by Figures A1-A10 in the Appendix. This means that the PPS method based on CSA as Jessen proposed and used for the Orange Objective Yield Survey is appropriate for within tree sampling of apples trees.

The graphs also indicate that simple

random sampling (SRS), which corresponds to $\gamma = 0$, produces a higher CV than for the PPS to CSA method. The reduction in CV's due to PPS sampling, as compared to SRS, is dependent on the cutoff CSA. For a cutoff CSA of 0.5 in², analysis indicates the PPS CV is about 1/3 of the SRS CV.

Figures 1 and 2 also indicate that the two stage method tends to produce lower CV's than the multiple stage method over all possible cutoff CSA's. This is more clearly shown in Figure 4, which is a cross section along $\gamma = 2$ from Figures 1 and 2. For cutoff CSA's in the 1 to 2 in² range, the two stage CV's are approximately 20% less than the multiple stage CV's.

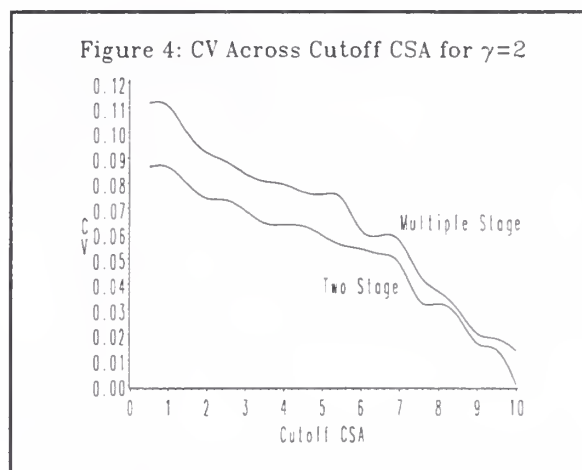
In the two stage process, the enumerator terminates the branch selection process after the second fork along the path. Branches at that point are terminal branches and will generally have a CSA much larger than the cutoff CSA. Two stage sampling is really multiple stage sampling at the first two forks. In the multiple stage approach, as long as two or more branches at each fork are greater than the cutoff CSA, branches continue to be sampled at each



stage. Consequently, more apples per tree are counted with the two stage approach for a comparable cutoff CSA than in a multiple stage approach, since all apples on all branches beyond the second fork are counted regardless of the branch size.

For cutoff CSA's in the 1 to 2 in² range, Figure 5 indicates the two stage approach involves counting 20-30 more apples per tree. Consequently, the two stage approach provides a lower CV but the cost is probably higher than a comparable multiple stage approach.

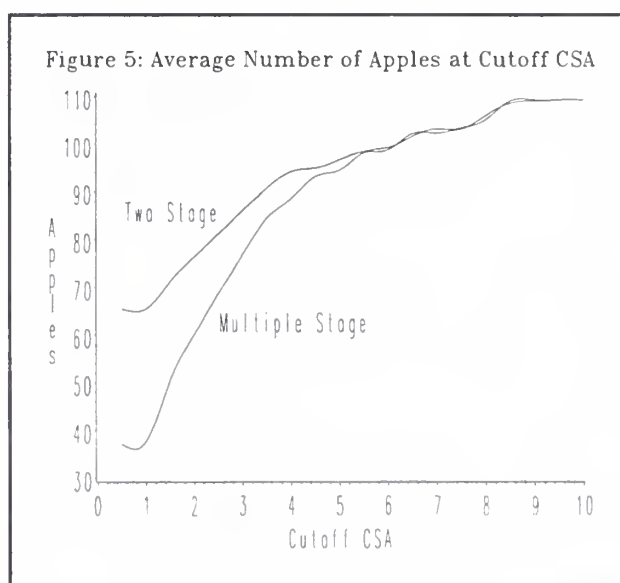
Relative per unit cost data will be required to find the optimal sampling technique. For the two stage technique the cost includes the time involved for: identifying, measuring, and sampling the primaries; identifying, measuring, and sampling second stage branches; and counting the fruit on the selected branch. The cost for the multiple stage approach does include the measuring of additional branches, but involves counting fewer apples. A cost analysis would evaluate the optimal cutoff for each approach and the optimal number of branches to sample at each stage relative



to CV and total cost. (All analyses in this report were based on sampling one branch at each stage, but multiple branches could be sampled depending on the cost efficiency.)

CONCLUSIONS

This study evaluates branch size measures of the form $x_i = r^\gamma$ where r =radius of branch i and $\gamma \geq 0$ for use in probability



proportional to size (PPS) sampling of apple tree branches to estimate the number of apples on a tree. Graphical analysis demonstrates that a size measure of r^2 will produce the lowest CV's regardless of the tree's variety, rootstock, age, or geographic region. This size measure produces the lowest CV's for both two stage and multiple stage random path sampling. Other NASS fruit tree surveys typically use the branch cross sectional area (CSA) as the size measure for PPS sampling. Since r^2 is proportional to the CSA ($CSA = \pi r^2$), this study also confirms that the CSA is an optimal size measure, in relation to CV, for PPS sampling. PPS sampling based on a size measure of r^0 is the same as simple random sampling (SRS). Consequently, this analysis shows that PPS sampling based on CSA produces lower CV's than SRS. However, PPS sampling does require measuring branches at each stage where SRS does not require any measurements, so there is additional cost involved with PPS sampling. A thorough cost analysis would determine the cost efficiency of each approach.

The graphical analysis also indicates that the two stage random sampling approach involves counting 20-30 more apples per tree, on average, than the multiple stage approach for cutoff CSA's in the 1 to 2 in² range. The resulting two stage CV's, using PPS sampling based on CSA, are approximately 20% less than the multiple stage CV's. Since a two stage approach is really a multiple stage approach with a relatively large cutoff CSA, cost analysis would first focus on an optimal cutoff CSA for a multiple stage application. If the

optimal cutoff CSA was 4 in² or more, then possibly the simple two stage approach would be attractive.

RECOMMENDATIONS

The next step in developing an Apple Objective Yield program is to conduct a cost efficiency analysis to determine the optimal within tree sampling approach. The objective of such a cost analysis should be to evaluate the trade-offs between costs and CV's. Within tree sampling costs are dependent on the amount of time involved in measuring and sampling branches at each stage and in the amount of time required to count the apples. The cost analysis would provide recommendations regarding:

- 1) PPS sampling vs. simple random sampling
- 2) the optimal cutoff CSA
- 3) the number of branches to sample at each stage.

A simple cost model could be developed based on "expert" estimates of the time involved to measure and sample branches at each stage and to count the apples on each terminal branch. A small field study could be conducted by the Ohio Applications Research Section to help develop or verify a cost model. The model could be applied to the 1991 pilot study data for a cost efficiency analysis.

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Table A1: Background Information on the Mapped Trees

Tree	Age	Radius of Stem (mm)	Variety	Rootstock
11	7	39.51	Redchief	Semi-dwarf
12	7	30.06	Redchief	Semi-dwarf
21	11	81.07	Golden Delicious	Standard
22	11	85.98	Golden Delicious	Standard
31	13	68.73	Newtown	Semi-dwarf
32	13	67.22	Newtown	Semi-dwarf
41	2	26.81	Gala	Semi-dwarf
42	2	20.27	Gala	Semi-dwarf
51	15	55.50	Redchief	Semi-dwarf
52	15	59.09	Redchief	Semi-dwarf
61	8	39.25	Granny Smith	Semi-dwarf
62	8	35.10	Granny Smith	Semi-dwarf
81	10	90.07	Early Brite	Semi-dwarf
82	10	51.67	Early Brite	Semi-dwarf
91	19	77.17	Oregonspur	Semi-dwarf
92	19	105.31	Oregonspur	Semi-dwarf
121	34	219.68	Golden Delicious	Standard
122	34	176.68	Golden Delicious	Standard
131	8	45.32	Redchief	Semi-dwarf
132	8	47.74	Redchief	Semi-dwarf
151	8	36.25	Redchief	Semi-dwarf
152	8	46.21	Redchief	Semi-dwarf
161	8	58.03	Grandspur	Semi-dwarf
162	8	59.95	Grandspur	Semi-dwarf
171	11	54.00	Oregonspur	Semi-dwarf
172	11	44.86	Oregonspur	Semi-dwarf
181	3	62.46	Fuji	Semi-dwarf
182	3	49.23	Fuji	Semi-dwarf
191	7	61.47	Redchief	Semi-dwarf
192	7	69.32	Redchief	Semi-dwarf
201	10	70.93	Golden Delicious	Semi-dwarf
202	10	70.9	Golden Delicious	Semi-dwarf
211	27	106.57	Bisbee	Semi-dwarf
212	27	87.17	Bisbee	Standard
221	11	47.96	Redchief	Semi-dwarf
222	11	59.26	Redchief	Semi-dwarf
231	23	65.36	Earlistripe	Standard
232	23	87.17	Earlistripe	Standard
241	14	37.37	Redchief	Semi-dwarf
242	14	52.85	Redchief	Semi-dwarf
271	27	116.42	Golden Delicious	Standard
272	27	120.75	Golden Delicious	Standard
281	8	123.27	Fuji	Semi-dwarf
282	8	125.75	Fuji	Semi-dwarf
291	18	105.79	Redspur	Standard
292	18	30.40	Redspur	Standard
301	26	178.41	Hiearly	Standard
311	23	130.56	Hiearly	Standard
312	23	136.10	Hiearly	Standard
321	11	42.27	Redchief	Semi-dwarf
322	11	40.53	Redchief	Semi-dwarf

APPENDIX

Figure A1: Multiple Stage for Red Delicious

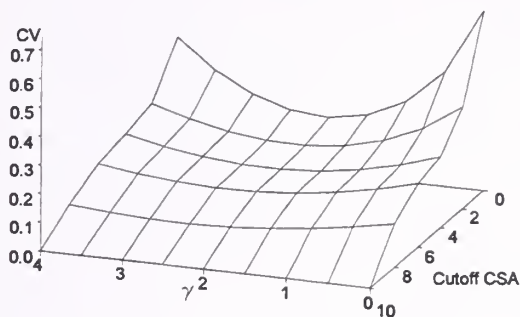


Figure A2: Multiple Stage for All Other Varieties

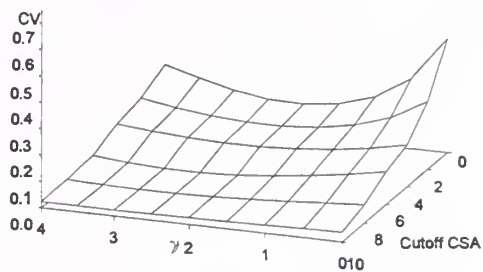


Figure A3: Multiple Stage for Standard Rootstock

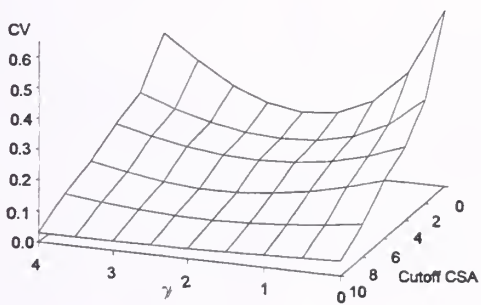


Figure A4: Multiple Stage for Semi-Dwarf Rootstock

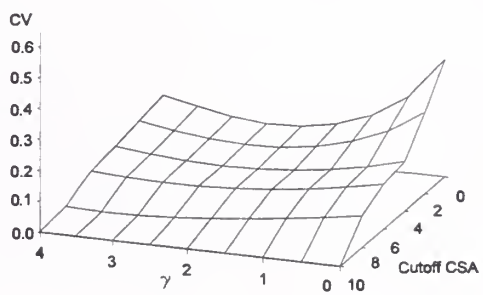


Figure A5: Multiple Stage for Trees 1 to 10 Years Old

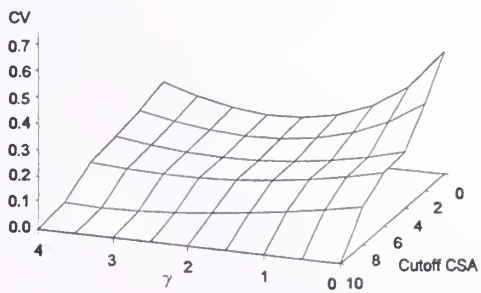
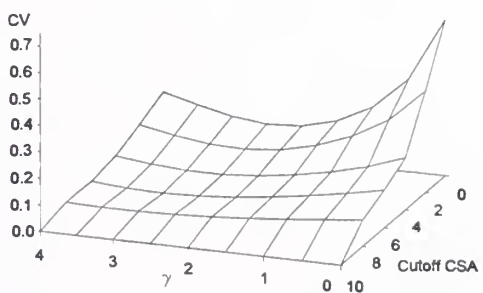


Figure A6: Multiple Stage for Trees 11 to 20 Years Old



APPENDIX

Figure A7: Multiple Stage for Trees Older than 20 Years

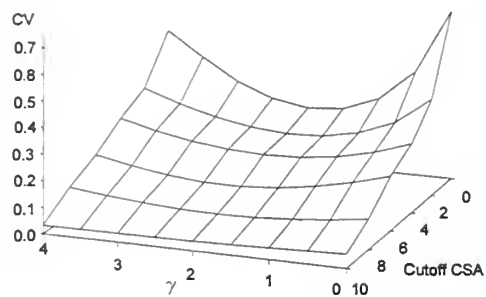


Figure A8: Multiple Stage for Yakima Region

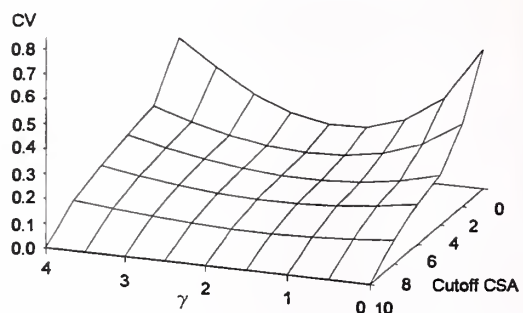


Figure A9: Multiple Stage for Columbia Basin Region

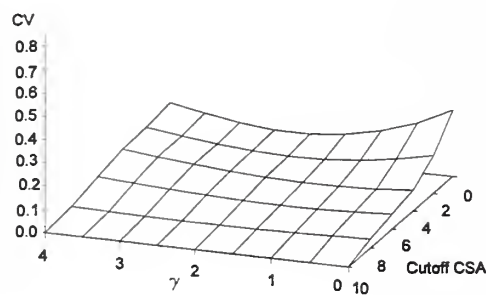


Figure A10: Multiple Stage for Wenatchee Region

